Edge Domination Number of Jump Graph

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Abstract

A Set $D \subseteq E[J(G]$ is dominating set of jump graph, if every edge not in D is adjacent to a edge in D. The domination number of the jump graph is the minimum cardinality of dominating set of jump graph J(G).We also study the graph theoretic properties of $\gamma'[J(G)]$ and its exact values for some standard graphs. The relation between $\gamma'[J(G)]$ with other parameter are also investigated.

Keywords: Edge Domination Number, Jump Graph, Diameter.

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1. INTRODUCTION

Let G (p,q) be a graph with p=$|V|$ and q=$|E|$ denote the number of vertices and edges of a graph G respectively. All the graphs considered here are finite, non-trivial, undirected and connected without loops or multiple edges.

In general the degree of vertex v in a graph G is the number of edges of G incident with v and it is denoted by degv. The maximum(minimum) degree among the vertices of G is denoted by $\Delta(G)(\delta(G))$.We denote the minimum number of edges in edge cover of G (i.e., edge cover number) by $\alpha_1(G)$ and the minimum number of edges in independent set of edges of G(i.e., edge independent set) by $\beta_1(G)$. The subgraph induced by $X \subseteq V$ is denoted by <X>. A vertex of degree one is called an pendent vertex. A vertex adjacent to pendent vertex is called the support vertex. The maximum d(u,v) for all u in G is eccentricity of v and the maximum eccentricity is the diameter diam(G). As usual P_n, C_n and K_n are respectively, the path, cycle and complete graph of order n, $K_{r,s}$ is the complete bipartite graph with two partite sets containing r and s vertices. Any undefined term or notation in this paper can be found in Harary [2] [4].

2. PRELIMINARY NOTES

The line graph L(G) of G has the edges of G as its vertices which are adjacent in G. We call the complement of line graph L(G) as the jump graph J(G) of G, found in [1]. The jump graph J(G) of a graph G is the graph defined on E(G) and in which two vertices are adjacent if and only if they are not adjacent in G. Since both L(G) and J(G) are defined on the edge set of a graph G, it follows that isolated vertices of G (If G has) play no role in line graph and jump graph transformation. We assume that the graph G under consideration is nonempty and has no isolated vertices [1].

Definition 2.1: We now define the edge domination number of jump graph. Let G=(V,E) be a graph. A set $D \subseteq E$ is said to be a dominating set, if every edge not in D is adjacent to a edge in D. The edge domination number of G, denoted by $\gamma^1(G)$, is the minimum cardinality of a dominating set. Analogously, a set $D \subseteq E[J(G)]$

ISSN 2229-5518

is said to be dominating set of J(G), if every edge not in D is adjacent to a edge in D. The domination number of jump graph, denoted by $\gamma^1[J(G)]$, is the minimum cardinality of a dominating set in J(G). For any graph G with p≤4,the jump graph J(G) of G,I s disconnected since we study only the connected jump graph, we choose p >4 [3]

2. MAIN RESULTS

Theorem 3. 1: **1.** For any path P_p, with p≥ 5, $\gamma'[J(P_p)]$ =2

2. For any Cycle C_p , with p≥ 5, $\gamma'[J(C_p)]$ =2

3. For any Complete graph K_p with p≥ 5 , $\gamma'[J(K_p)]$ =3

4. For any complete bipartite graph K_{mn}.

$$\gamma'[J(K_{m,n})] = \begin{cases} 2 & for\ k_{2,n}\ where\ n > 2 \\ 3 & for\ k_{m,n}\ where\ m,n \geq 3 \end{cases}$$

5. For any wheel W_p $\gamma'[J(W_p)] = \begin{cases} 3\ for\ p = 5,6 \\ 2\ \ for\ p \geq 7 \end{cases}$

Theorem 3.2: For any connected graph G $\gamma'[J(G)]$ ≥ 2

Proof of the theorem is obvious

The following theorem gives the relationship between edge domination number of a graph with its edge domination number of a jump graph

Theorem 3.3: For any connected graph G with diameter, diam(G) ≥2, $\gamma'[J(G)]$ ≥ 2

Proof: Let uv be a path of maximum distance in G. Then d(u,v) = diam(G)

We can prove the theorem with the following cases

Case1: For diam(G) =2,Choose a vertex v_1 of eccentricity 2 with maximum degree among others. Let E_1 = { e_1^1, e_2^1 ...} corresponding to the elements of $\{v_1, v_2, ...\}$ forming a dominating set in jump graph J(G).Every edge $v \notin E_1$ is adjacent to a edge in E_1. Hence $E_1$is a minimum dominating set . So the edge domination number of the jump graph is $\gamma'[J(G)]$ >2.

Case2: For diam(G)>2,let v_1 be any vertex adjacent to v and v_2 be any vertex adjacent to u. Let $\{v_1\ v_2\}$ $\subseteq$V(G) form a corresponding edge set { e_1^1, e_2^1 }$\subseteq$ E(J(G)).These two edges form a dominating set in jump graph. Since these edges { e_1^1, e_2^1 } are adjacent to all other edges of E(J(G)),it follows that $\{e_1,e_2\}$ becomes a minimum dominating set. Hence $\gamma'[J(G)]$ =2.

In view of above cases, we can conclude that for any connected graph G $\gamma'[J(G)]$ ≥ 2

Theorem 3.4: For any tree T with diameter greater than 3, $\gamma'[J(T)]$ = n

Proof: If the diameter is less than or equal to 3, then the jump graph will be disconnected.

Let uv be a path of maximum length in a tree T where diameter is greater than 3.Let e_i be the pendent vertex adjacent to u and e_k be the pendent edge adjacent to v. The edge set e_i ,i=1,2,3,…n of J(T) corresponding to the vertices in T will form the dominating set in J(T).Since all the other edges of E[J(T)] are adjacent with e_i i=1,2,3…n it form a minimum dominating set. Hence $\gamma'[J(T)]$ = n

Theorem 3.5: For any connected (p,q) graph G, $\gamma^1[J(G)] \leq p - \Delta(G)$ where $\Delta(G)$ is the maximum degree of G.

Proof: Let $E = \{e_1, e_2, ..., e_n\}$ be the set of edges in G and let $E_1 = E - e_1$ where e_1 is one of the edge with maximum degree. By definition of jump graph, E(G) = V[J(G)] . Consider $I = \{v_1, v_2, ..., v_n\}$ as the set of vertices adjacent to e_1 in G. Let $H \subseteq E(J(G))$ be the set of edges of J(G) such that $H \subseteq V - I$. Then H itself forms a minimally dominating set. Therefore $\gamma^1[J(G)] \leq |V| - |I|$.Hence

$$\gamma^1[J(G)] \leq p - \Delta(G)$$

Theorem 3.6: For any connected (p,q) graph G, $2 \leq \gamma^1[J(G)] \leq \lfloor p/2 \rfloor$

Proof: An edge $\{e_i\}$is any connected graph G is adjacent to atleast one more edge in G. In jump graph, the vertex $\{e_1^|\}$corresponding to $\{v_i\}$ is non adjacent to $\{e_i^k, e_j^j\}$ of $\{v_k, v_j\}$ in J(G). Therefore by definition of edge dominating number of graph $\gamma^1(G)$, the dominating set contains atleast two elements. Hence $\gamma^1[J(G)] \geq 2 \rightarrow (1)$

Let E by the set of edges in G. Then E=V[J(G)]. Suppose $D = \{e_1, e_2, ..., e_k\}$ be the dominating set. Then E-D is also a dominating set. One among these two sets will form a minimal dominating set. So by the definition of edge domination number of graph, we can say edge domination number $\gamma^1[J(G)]$ of jump graph is given by $\gamma^1[J(G)] \leq \min\{|D|, |E = D|\} \leq \lfloor 1/2 \rfloor \rightarrow (2)$

from (1) and (2) we get $2 \leq \gamma^1[J(G)] \leq \lfloor p/2 \rfloor$.

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